Exercise 3.1

The kinematic wave

This document provides an outline for the solution of Exercise 3.1 provided in the book *Wave Propagation in Fluids*, author V. Guinot, Publisher ISTE.

1. Problem

Consider the rectangular channel used in Exercise 2.5 (see Subsection 2.7.2.5). The flow is assumed to obey Strickler's friction law [1.81]. Steady state is assumed.

1) The initial water depth is uniformly equal to $h_0 = 1$ m. Assuming that the wide channel approximation is applicable, compute the initial discharge in the canal under the assumption of a uniform, steady flow ($S_0 = S_f$). Provide the expression of the celerity for the kinematic wave. Carry out the numerical application for the parameters in Table 2.1.

2) A perturbation $\Delta h = 0.5$ m appears instantaneously at the upstream end of the channel. Show that a shock wave appears. Provide the expression of the propagation speed of the shock wave. Carry out the numerical application for the parameters in Table 2.1.

Symbol	Meaning	Value
b	Channel width	10 m
g	Gravitational acceleration	9.81 m/s^2
K_{Str}	Strickler coefficient	40
S_0	Channel bottom slope	0.1 %, 1%, 5%

Table 2.1. *Parameters for Exercise 2.5.*

2. Solution

2.1. *Question 1*

The discharge Q and the celerity λ are given by

$$\left.\begin{aligned} Q &= K_{\mathrm{Str}} S_0^{1/2} b h^{5/3} \\ \lambda &= \frac{5}{3} K_{\mathrm{Str}} S_0^{1/2} h^{2/3} \end{aligned}\right\} \quad [1]$$

From Table 2.5, one has
− for $S_0 = 10^{-3}$: $Q = 12.7$ m^3 s^{-1}, $\lambda = 2.1$ m s^{-1} ;
− for $S_0 = 10^{-2}$: $Q = 40$ m^3 s^{-1}, $\lambda = 6.7$ m s^{-1} ;
− for $S_0 = 5.10^{-2}$: $Q = 89.4$ m^3 s^{-1}, $\lambda = 14.9$ m s^{-1} ;

2.2. *Question 2*

The celerity of the kinematic wave in a rectangular channel is an increasing function of the water depth (see Eq. [1]). Using $h = 1.5$ m leads to the following values :
− for $S_0 = 10^{-3}$: $\lambda = 2.76$ m s^{-1} ;
− for $S_0 = 10^{-2}$: $\lambda = 8.$ m s^{-1} ;
− for $S_0 = 5.10^{-2}$: $\lambda = 19.5$ m s^{-1} ;

In all cases the celerity on the left-hand side is larger than the celerity on the right-hand side. Consequently, a shock appears, as sketched in Figure 1.

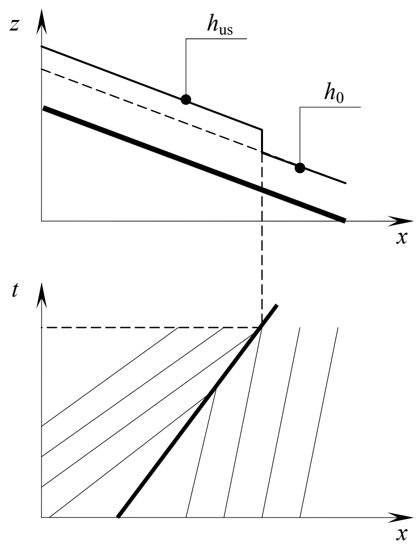

Figure 1. *Kinematic wave. Shock created by an increase in the water depth. Representation in the physical space (top) and in the phase space (bottom).*

The speed of the shock is given by

$$c_s = \frac{Q_{\mathrm{us}} - Q_0}{A_{\mathrm{us}} - A_0} = K_{Str} S_0^{1/2} \frac{h_{\mathrm{us}}^{5/3} - h_0^{5/3}}{h_{\mathrm{us}} - h_0} \quad [2]$$

where A is the cross-sectional area and the subscripts 0 and us denote the initial and upstream condition respectively. The numerical values in Table 2.5 lead to the following numerical values for c_s

− for $S_0 = 10^{-3}$: $c_s = 2.44$ m s^{-1} ;
− for $S_0 = 10^{-2}$: $c_s = 7.7$ m s^{-1} ;
− for $S_0 = 5.10^{-2}$: $c_s = 17.3$ m s^{-1} ;

Note that the shock speed c_s lies between the celerity on both sides of the shock, as mentioned in Chapter 3.