Exercise 1.2

The kinematic wave equation

This document provides an outline for the solution of Exercise 1.2 provided in the book *Wave Propagation in Fluids*, author V. Guinot, Publisher ISTE.

1. Problem

Derive the analytical formula for the celerity λ in a channel, the cross-section of which is a trapezium of bottom width b_0, the embankments of which make an angle θ with the vertical (Figure 1.25). Provide a graphical representation of λ as a function of the water depth h and carry out the numerical application for the set of parameters provided in Table 1.3. Note that the case $b_0 = 0$ corresponds to a triangular channel, in which case only positive values of θ are meaningful.

Symbol	Meaning	Value
b_0	Channel bottom width	0 m, 10 m, 40 m
K_{Str}	Strickler coefficient	40
S_0	Channel bed slope	1%
θ	Angle between the embankments and the vertical	–30°, 0°, 30°, 60°

Table 1.3. *Parameters for Exercise 1.2.*

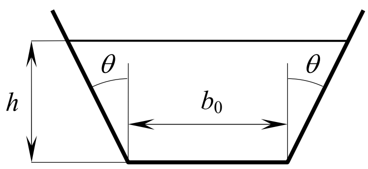


Figure 1.25. *Definition sketch for the trapezoidal channel.*

2. Solution

The celerity is given by the first equation [1.86], recalled hereafter

$$\lambda = \frac{\partial Q}{\partial A}$$

Note that Q is given as in Eq. [1.83], recalled hereafter

$$Q = K_{Str} \frac{A^{5/3}}{\chi^{2/3}} S_0^{1/2}$$

The wetter perimeter being a complex function of A, the expression of λ is more easily derived by introducing the water depth h in the derivations. Indeed,

$$\lambda = \frac{\partial Q}{\partial h}\frac{\partial h}{\partial A} = \frac{\partial Q}{\partial h}\left(\frac{\partial A}{\partial h}\right)^{-1} \qquad [1]$$

The cross-sectional area A and the wetted perimeter χ are easily expressed as functions of h

$$\left.\begin{aligned} \chi &= b_0 + \frac{2h}{\cos\theta} \\ A &= (b_0 + h\tan\theta)h \end{aligned}\right\} \qquad [2]$$

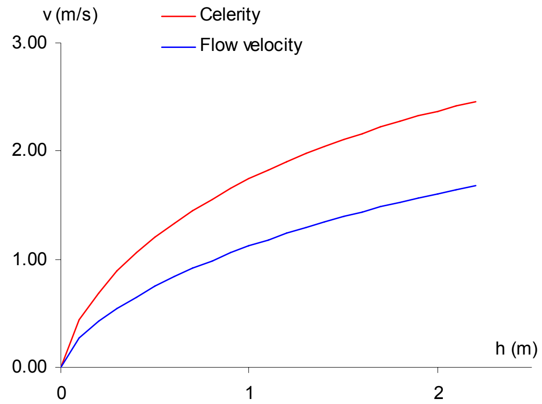
Substituting Eq. [2] into Eq. [1.83] leads to the following expression for Q

$$Q = K_{\mathrm{Str}} S_0^{1/2} \frac{[(b_0 + h\tan\theta)h]^{5/3}}{\left(b_0 + \dfrac{2h}{\cos\theta}\right)^{2/3}} \qquad [3]$$

Differentiating Eqs. [2] andt [3] with respect to h, substituting the derivatives of A et Q into Eq. [1] yields

$$\left.\begin{aligned} \frac{\partial A}{\partial h} &= b_0 + 2h\tan\theta \\ \frac{\partial Q}{\partial h} &= \frac{K_{\mathrm{Str}} S_0^{1/2}}{3}\left[\frac{(b_0 + h\tan\theta)h}{b_0 + \dfrac{2h}{\cos\theta}}\right]^{2/3} \\ &\quad \times \left(5(b_0 + 2h\tan\theta) + 4\frac{(b_0 + h\tan\theta)h}{b_0\cos\theta + 2h}\right) \end{aligned}\right\} \qquad [4]$$

The variations in λ are sketched in the graph for $b_0 = 10$ m, $S_0 = 10^{-3}$ et $\theta = 0$. Also see the spreadheet http://vincentguinot.free.fr/waves/ex12.xls.


v (m/s)
Celerity
Flow velocity
3.00
2.00
1.00
0.00
h (m)
0
1
2