Exercise 2.1

Water hammer

This document provides an outline for the solution of Exercise 2.1 provided in the book *Wave Propagation in Fluids*, author V. Guinot, Publisher ISTE.

1. Problem

Consider a horizontal pipe of cross-sectional area A, where the pressure waves propagate at the speed c. Friction is assumed to be negligible. The initial flow conditions are steady state conditions, with a pressure and velocity uniformly equal to p_0 and u_0 respectively.

A variation Δp in the pressure appears at the left-hand end of the pipe and propagates to the right at the speed c (Figure 2.22). As a consequence, a variation Δu appears in the flow velocity.

1) Show that Δp and Δu verify the following relationship

$$\Delta p = \rho c \Delta u \qquad [2.233]$$

2) Assume now that the wave propagates from right to left. Show that the following relationship holds between Δp and Δu

$$\Delta p = -\rho c \Delta u \qquad [2.234]$$

These equations are called Joukowski's relationships.

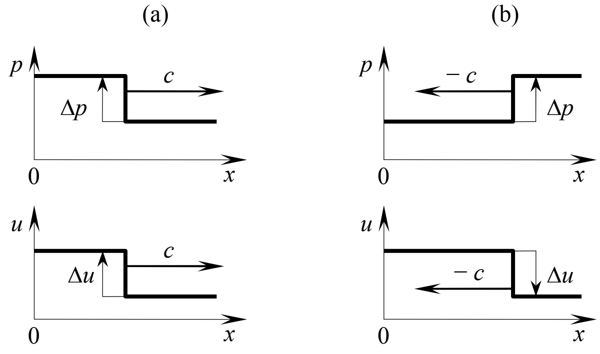

Figure 2.22. *Propagation of a pressure and velocity variation in a pipe. Propagation from left to right (a), from right to left (b).*

2. Solution

2.1. *Question 1*

Tye characteristic equations [2.82] are used across the pressure discontinuity. Since the pressure discontinuity moves at the celerity $+ c$, the negative characteristic $\mathrm{d}x/\mathrm{d}t = - c$ must be used (Figure 1). The characteristic $\mathrm{d}x/\mathrm{d}t = - c$ connects the point A (p_0, Q_0) to the point B ($p_0 + \Delta p$, $Q_0 + \Delta Q$).

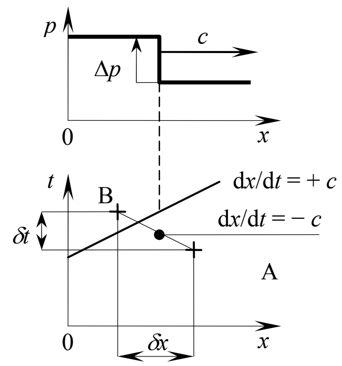

Figure 1. *Using the characteristic form across a pressure wave.*

The first equation [2.81] yields

$$p_0 - \rho c u_0 = p_0 + \Delta p - \rho c(u_0 + \Delta u) + \int_A^B \left(\frac{kc}{A}|u|u + \rho g c \sin\theta\right) dt \qquad [1]$$

The points A and B on each side of the discontinuity can be made arbitrarily close to each other. When the time and space intervals δt and δx between these two points tend to zero, equation [1] becomes

$$\Delta p = \rho c \Delta u \qquad [2]$$

2.2. *Question 2*

The reasoning is exactly the same as in the previous question, except that the characteristic $+ c$ must be used instead of the characteristic $- c$.