Exercise 2.7

The Euler equations

This document provides an outline for the solution of Exercise 2.5 provided in the book *Wave Propagation in Fluids*, author V. Guinot, Publisher ISTE.

1. Problem

A loudspeaker may be schematized as a plane membrane of cross-sectional area A subjected to a displacement in the direction x normal to the plane (Figure 2.26).

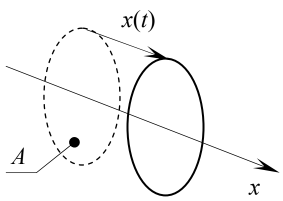

Figure 2.26. *Definition sketch of a loudspeaker membrane.*

Both sides of the membrane are in contact with the ambient air (orifices on the rear side of the cabinet to allow for such a contact with the back side of the membrane). Assume that the movement $x(t)$ of the membrane can be described by a periodic, sinusoidal function of time in the form

$$x(t) = a\cos(2\pi Nt) \qquad [2.235]$$

where a is the (constant) amplitude of the movement and N is the (constant) frequency of the sound signal.

1) Assuming a constant speed of sound c, provide the expression for the pressure as a function of time on both sides of the membrane.

2) Determine the average mechanical power needed to move the membrane over a period. Show that the power is proportional to the square of the frequency.

3) Carry out the numerical application for the parameter values in Table 2.3.

Symbol	Meaning	Value
A	Surface of the membrane	0.2 cm^2, 700 cm^2
a	Amplitude of the membrane displacement	0.1 mm, 5 mm
N	Frequency of the signal	20 Hz, 440 Hz, 2kHz, 16 kHz
p_0	Atmospheric pressure (for air at rest)	10^5 Pa
γ	Polytropique constant for perfect gases	1.4
ρ_0	Air density (at rest)	1.2 kg/m^3

Table 2.3. *Parameters for Exercise 2.7.*

2. Solution

2.1. *Question 1*

The membrane transmits its own displacement to the neighbouring air molecules. The velocity u of the gas next to the membrane is given by

$$u(t) = 2\pi Na\cos(2\pi Nt) \qquad [1]$$

The left-hand side of the membrane is connected to the undisturbed, initial state by a characteristic dx/dt = + c_0 (Figure 1), while the right-hand side of the membrane is connected to the undisturbed state by a characteristic dx/dt = – c_0.

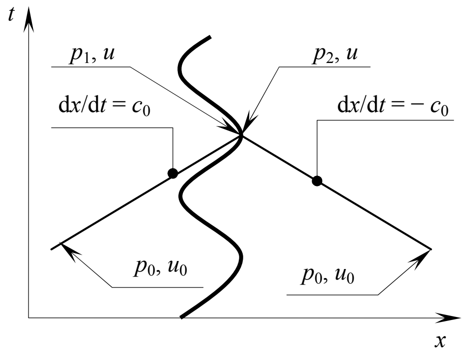

Figure 1. *Definition sketch for the location of the membrane (bold line) and the characteristic lines (thin lines) in the phase space.*

Denoting by p_1 and p_2 the pressure on the left- and right-hand side of the membrane respectively, the following relationships may be derived from Eqs. [2.205]

$$\left.\begin{aligned} p_1 + \rho_0 c_0 u &= p_0 + \rho_0 c_0 u_0 \\ p_2 - \rho_0 c_0 u &= p_0 - \rho_0 c_0 u_0 \end{aligned}\right\} \qquad [2]$$

where $u_0 = 0$ because the air is initially at rest. The force F exerted by the membrane on the air molecules is

$$F = (p_2 - p_1)A = 2\rho_0 c_0 u = 4\pi ANa\rho_0 c_0 \cos(2\pi Nt) \qquad [3]$$

2.2. *Question 2*

The power $P(t)$ is equal to the product Fu :

$$P(t) = 8\rho_0 c_0 A[\pi Na\cos(2\pi Nt)]^2 \qquad [4]$$

Considering that the average of the function $\cos^2$ is ½, the average power $\overline{P}$ over a complete period is

$$\overline{P} = 4\rho_0 c_0 A(\pi Na)^2 \qquad [4]$$

The average power is proportional to the square of both the frequency and amplitude.

2.3. *Question 3*

Note that the sound speed is given as in Eq. [2.207] :

$$c_0 = \left(\gamma \frac{p_0}{\rho_0}\right)^{1/2}$$

Configuration	Frequency (Hz)	Maximum force (N)	Average power (W)
Phone loudspeaker	20	2.1×10^{-4}	1.3×10^{-6}
Phone loudspeaker	440	4.5×10^{-3}	6.3×10^{-4}
Phone loudspeaker	2000	2.6×10^{-2}	1.3×10^{-2}
Phone loudspeaker	16000	1.65×10^{-1}	8.3×10^{-1}
Loudspeaker stack	20	7.2	4.5×10^{-1}
Loudspeaker stack	440	1.6×10^{2}	2.2×10^{2}
Loudspeaker stack	2000	7.2×10^{2}	4.5×10^{3}
Loudspeaker stack	16000	5.8×10^{3}	2.9×10^{5}

Note that these are only indicative values. In practice, the shape of the loudspeaker membrane, the wiring, etc. influence the final figures.