Exercise 5.2

Visual assessment of the Mach number

This document provides an outline for the solution of Exercise 5.2 provided in the book *Wave Propagation in Fluids*, author V. Guinot, Publisher ISTE.

1. Problem

Airplanes entering high moisture regions at supersonic speeds sometimes generate condensation patterns that develop next to the convex part of the wings and the hull. A condensation pattern indicates a sudden pressure drop, which is an indication that a shock wave is present (Figure 5.13).

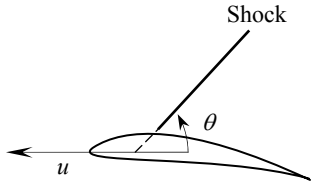

Figure 5.13. *Condensation zone developing along the shock wave for a plane in supersonic flight.*

Show that the angle between the shock and the velocity vector of the airplane is given by

$$M = \frac{1}{\sin\theta} \qquad [5.86]$$

which allows the Mach number M to be determined visually.

2. Solution

In what follows, the airplane is represented as a moving point for the sake of clarity. The shock wave generated by the movement of the airplane is the envelope of the domains of influence at successive times materialized by the circles in Figure 1. The domains of influence are represented in a coordinate system attached to the plane.

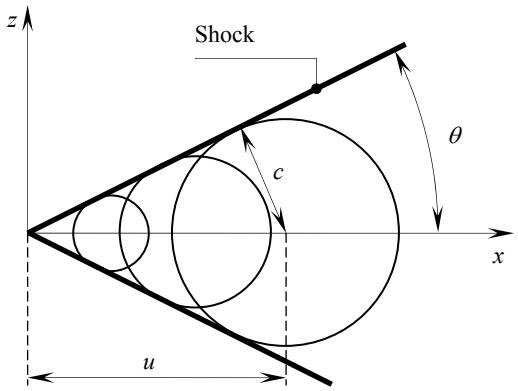

Figure 1. *Domain of influence at successive times.*

The domain of influence is a cone, the radius of which increases at the speed c, while the centre moves a the speed u. The ratio c/u is equal to the sine of the angle θ between the shock and the longitudinal coordinate. Introducing the Mach number $M = u/c$ leads to Eq. [5.85].