# Exercice 6.1 (4)

# Finite difference methods for scalar laws

This document provides an outline for the solution of Exercise 6.1 provided in the book *Wave Propagation in Fluids*, author V. Guinot, Publisher ISTE. The application to Exercise 1.4 is detailed hereafter.

**1. Problem**

Check the conclusions of Exercises 1.1 to 1.5 using finite difference methods. The following methods are advised:

– a characteristic-based method,

– an upwind scheme (conservative version),

– Preissmann's scheme,

– a TVD scheme.

**2. Solution**

**2.1.** ***Discretization***

Only the conservative version of the upwind scheme is implemented because it allows weak solutions to be dealt with without any particular treatment. The Buckley-Leverett equation is discretized as follows

$$s_i^{n+1} = s_i^n + \frac{\Delta t}{\Delta x_{i-1/2}}\left(F_{i-1/2}^{n+1/2} - F_{i+1/2}^{n+1/2}\right) \quad [1]$$

where $F$ is defined as in Eq. [1.111] :

$$F_{i-1/2}^{n+1/2} = \frac{\left(s_{i-1/2}^{n+1/2}\right)^2}{\left(s_{i-1/2}^{n+1/2}\right)^2 + \left(1 - s_{i-1/2}^{n+1/2}\right)^2 b_{\mathrm{BL}}} V_d \quad [2]$$

with the cell size $\Delta x_{i-1/2}$ defined as

$$\Delta x_{i-1/2} = \frac{x_{i+1} - x_{i-1}}{2} \quad [3]$$

The saturation $s_{i-1/2}^{n+1/2}$ is approximated as

$$s_{i-1/2}^{n+1/2} = \begin{cases} s_i^n & \text{si } V_d < 0 \\ s_{i-1}^n & \text{si } V_d \geq 0 \end{cases} \quad [4]$$

Note that the explicit upwind scheme is subjected to a stability constraint. The reader is invited to derive the expression for this constraint.

## 2.2. *Application*

The discretization is applied with the following parameters (see spreadsheet http://vincentguinot.free.fr/waves/ex614.xls) :

–Darcy velocity $V_d$ = 1 m/day ;

– shape parameter $b_{BL}$ = 1 ;

– cell size $\Delta x$ = 1 m ;

– time step $\Delta t$ = 0,5 day.

For the sake of clarity, only positive Darcy velocities have been considered in the implementation. The saturation is prescribed at the left-hand boundary.

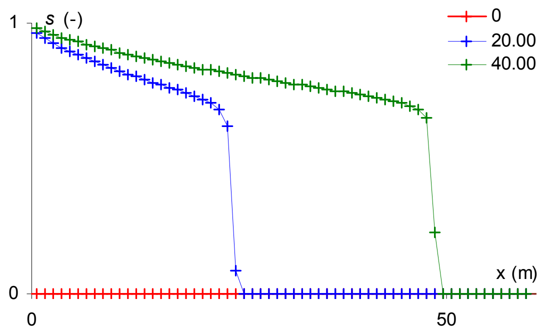


**Figure 1.** *Saturation profile at two different times.*