Exercise 1.5

Transport with adsorption/desorption

This document provides an outline for the solution of Exercise 1.5 provided in the book *Wave Propagation in Fluids*, author V. Guinot, Publisher ISTE.

1. Problem

Consider an aquifer of length L initially contaminated by a solute with an initial concentration C_0. The solute is adsorbed on the soil grains following Langmuir's adsorption law, the parameters of which can be found in Table 1.6.

Symbol	Meaning	Value
C_0	Initial concentration in the liquid phase	1.5 kg/m^3
C_L	Maximum adsorbed mass concentration	10^{-4} g/g
k_L	Constant in Langmuir's law	5 l/g
L	Length of the aquifer	200 m
V	Darcy velocity	1 m/day
θ	Water content (porosity) in the aquifer	0.25
ρ_A	Soil bulk density	1500 kg/m

Table 1.6. *Parameters for Exercise 1.6.*

The aquifer is to decontaminated by injecting pure water at the left-hand boundary of the aquifer starting at $t = 0$ (Figure 1.28). The Darcy velocity is denoted by V. The approximation [1.141] is assumed to be valid.

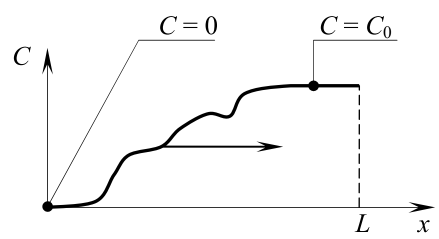

Figure 1.28. *Decontaminating an aquifer by injecting pure water.*

1) Determine the expression and the numerical value of the time T_d at which the contaminant concentration starts to decrease at the right-hand boundary of the aquifer ($x = L$).

2) Determine the expression for the profile for $t > 0$ (searching tip: try to express x as a function of C_T as opposed to the more classical approach $C_T(x)$).

2. Solution

It is first noticed that the wave celerity is an increasing function of the concentration. Indeed, assuming that the approximation [1.141] is valid, using Eq. [1.125] for Langmuir's model leads to

$$R_F = 1 + \frac{\rho_A C_A}{\theta C_T} = 1 + \frac{\rho_A k_L C_L}{(1 + k_L C_T)\theta} \qquad [1]$$

The retardation factor is a decreaseing function of the transported concentration C_T . This could be expected because the number of available adsoption sites decreases as the concentration increases. In the (idealized) limit case where the transported concentration C_T is infinite, all the adsorption sites are occupied and the adsorbed quantity represents an infinitesimal fraction of the total contaminant in the system. The retardation factor tends to unity and the contaminant is transported at the speed of the flow.

As an immediate consequence, a contaminant with a large concentration is transported faster than a contaminant with a small concentration. Decontaminating an aquifer by injecting pure water leads to the spreading of the concentration profile because the tail of the concentration wave is slower than the head.

2.1. *Question 1*

The time T_L at which the decontamination front reaches $x = L$ is given by

$$T_L = \frac{L}{\lambda(C_0)} \qquad [2]$$

Using Eqs. [1.141] and [1] for the retardation factor leads to

$$T_L = \frac{L\theta}{V} R_F = \left(\theta + \frac{\rho_A k_L C_L}{1 + k_L C_0}\right)\frac{L}{V} \qquad [3]$$

The data in the Table yield a value of 67 days (see spreadsheet). Note that pure water covers the distance within 50 days (value obtained with $R_F = 1$).

2.2. *Question 2*

The concentration profile in the aquiefer is more easily determined by expressing x as a function of C. Using the same reasoning as in Exercise 1.4 leads to the following expression

$$\left.\begin{array}{ll} x(C) = \lambda(C)t & \text{for } 0 < C < C_0 \\ C = C_0 & \text{for } x > \lambda(C_0)t \end{array}\right\} \quad [4]$$

The calculation is implemented in the spreadsheet http://vincentguinot.free.fr/ondes/ex15.xls.