Exercise 2.5

The Saint Venant equations

This document provides an outline for the solution of Exercise 2.5 provided in the book *Wave Propagation in Fluids*, author V. Guinot, Publisher ISTE.

1. Problem

Consider a rectangular channel, the width and slope of which are denoted by b and S_0 respectively. The Strickler coefficient is assumed to be uniform. Steady, uniform flow is assumed, that is, the slope of the energy line is assumed to be identical to the slope of the bottom of the channel.

1) Provide the expression of the celerity λ of the kinematic wave as a function of the water depth h. The wide channel approximation ($h << b$) will be assumed in the calculation of the hydraulic radius for the sake of simplicity.

2) Provide the expressions of the celerities $\lambda^{(1)}$ and $\lambda^{(2)}$ of the waves in the Saint Venant equations as a function of h, assuming that the assumption of a steady, uniform flow and the wide channel approximation remain valid (the assumption of a uniform flow allows the flow velocity u to be expressed as a function of h).

3) Compare the two expressions and plot the wave celerities as functions of h for the numerical values provided in Table 2.1. Conclude about the validity of the kinematic wave approximation in practical applications.

Symbol	Meaning	Value
b	Channel width	10 m
g	Gravitational acceleration	9.81 m/s^2
K_{Str}	Strickler coefficient	40
S_0	Channel bottom slope	0.1 %, 1%, 5%

Table 2.1. *Parameters for Exercise 2.5.*

2. Solution

The celerity l_C of the kinematic wave in a rectangular channel under the wide channel assumption is available from Exercise 1.2

$$\lambda_C = \frac{5}{3}u = \frac{5}{3}K_{Str}S_0^{1/2}h^{2/3} \qquad [1]$$

where u is the flow velocity. The celerities $\lambda^{(1)}$ et $\lambda^{(2)}$ for the Saint Venant equations are given by

$$\left.\begin{aligned} \lambda^{(1)} &= u - c = K_{Str}S_0^{1/2}h^{2/3} - (gh)^{1/2} \\ \lambda^{(2)} &= u + c = K_{Str}S_0^{1/2}h^{2/3} + (gh)^{1/2} \end{aligned}\right\} \qquad [2]$$

The behaviour of the wave celerities is illustrated by Figure 1 for $S_0 = 10^{-3}$, 10^{-2} and 5×10^{-2}.

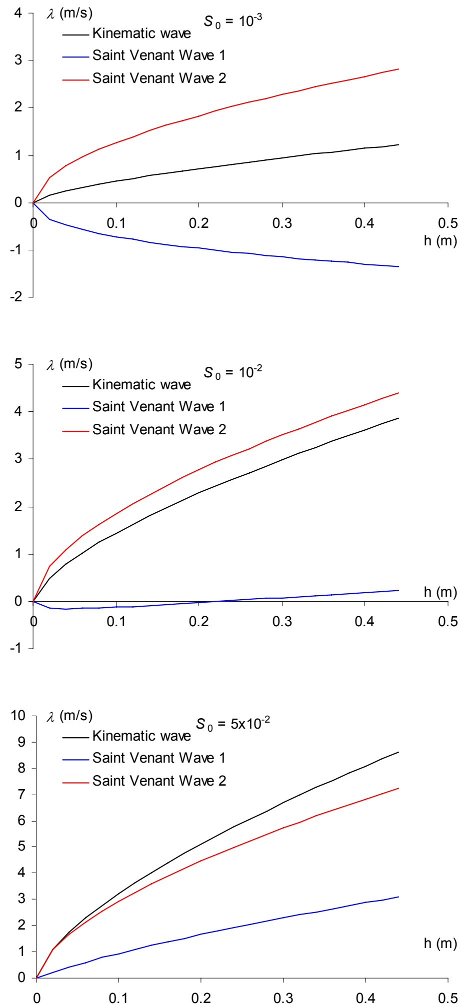

Figure 1. *Wave celerities as a function of h for various values of the bed slope.*

The kinematic wave provides a fairly accurate approximation of the wave $u + c$ in the Saint Venant equations (that is, the faster of the two waves), provided that the bottom slope is steep enough. With a slope $S_0 = 10^{-3}$, the celerity of the kinematic wave becomes equivalent to that of the wave $u + c$ when h is about 50 to 100 metres (Figure 2), which is not realistic.

Note that if uniform flow is assumed, there always exists a water depth above which the flow regime becomes supercritical. The reader may be interested in deriving the expression for this 'critical depth'.

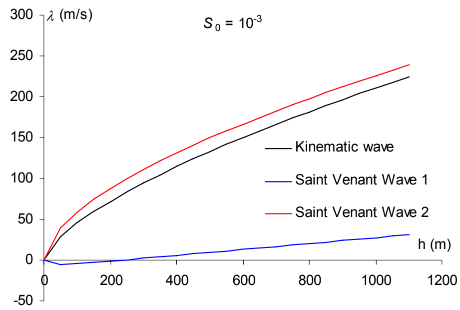

Figure 2. *Wave celerities as a function of h for* $S_0 = 10^{-3}$.